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Interaction between a large-scale structure and near-wall structures in channel flow

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Direct numerical simulation of a turbulent channel flow in a periodic domain of relatively wide spanwise extent, but minimal streamwise length, is carried out at Reynolds numbers $Re_\tau = 137$ and 349. The large-scale structures previously observed in studies of turbulent channel flow using huge computational domains are also shown to exist even in the streamwise-minimal channels of the present study. Moreover, the limitation of the streamwise length of the domain enforces the interaction between large-scale structures and near-wall structures, which consequently makes it tractable to extract a simple cycle of processes sustaining the structures in the present channel flow. It is shown that the large-scale structures are generated by the collective behaviour of near-wall structures and that the generation of the latter is in turn enhanced by the large-scale structures. Hence, near-wall and large-scale structures interact in a co-supporting cycle.

1. Introduction

The discovery of a self-sustaining process for certain so-called near-wall structures is a significant result from recent studies of wall-bounded turbulent flows. This process may be summarized as follows. In the vicinity of the wall, including the viscous and buffer layers which together constitute the near-wall region, a pair of streamwise vortices induces a low-speed streaky region, or ‘wall streak’, the instability of which makes the near-wall region energetic, and regenerates the streamwise vortices. This cyclic process, which was first recognized as autonomous in plane Couette flow in a minimal flow unit by Hamilton, Kim & Waleffe (1995) and Waleffe (1997), is thought to be fundamental in wall-bounded shear flows. Its relevance has been established by direct numerical simulation (DNS) of filtered flows (Jiménez & Simens 2001), the discovery of travelling-wave solutions (Waleffe 1998, 2001, 2003; Itano & Toh 2001) and time-periodic solutions (Kawahara & Kida 2001; Toh & Itano 2003) of the Navier–Stokes equations. In fact, turbulent intensity in the near-wall region obtained from DNS with huge computational domains is realized even in the minimal flow unit. This flow unit, however, contains only the near-wall structures; no other large-scale structures can exist. This indicates that the self-sustaining process is the fundamental process in the near-wall region.

As has been previously established by Kline *et al.* (1967), for example, the mean length of interval between wall streaks in the spanwise direction (which we term the mean spacing of the wall streaks) scales with wall units, and is around 100 wall units. It is also shown by Jiménez & Moin (1991) that turbulence cannot be sustained in DNS if the spanwise period of the computational domain is less than 100 wall units. It may thus also be expected that the near-wall structures themselves, which consist of a wall streak and a pair of quasi-streamwise vortices associated with the wall streak, scale with wall units. As the Reynolds number increases, these structures are restricted to the thin near-wall region, while almost all the rest of the domain becomes substantially covered by a region consisting of the upper region of the log layer together with the centre region. Hereinafter, we shall refer to this region as the outer region. However, in the outer region, turbulent intensity obtained from DNS with huge computational domains is not as small as that in minimal flow units. Hence, we would expect there to be another mechanism which could explain why turbulent intensity is relatively large in the outer region.

One possible phenomenon which may explain the unknown source of turbulent intensity in the outer region is the existence in this region of streaky structures of high and low streamwise velocity. A considerable amount of work has been done on this in recent years. Since these streaky structures scale with the outer length, for example, half the channel width, they are called large-scale structures or motions, and were first recognized by Miyake, Kajishima & Obana (1987) and Lee & Kim (1987) in plane Couette flow. So, turbulent intensity in the outer region consists not only of three-dimensional fine-scale turbulent fluctuation, but also of relatively coherent large-scale flows. Moreover, it is surprising that the streaky structures tend to maintain their location in the spanwise ordinate (which we will hereinafter refer to as the spanwise immobility of these structures) and elongate in the flow direction to fill the computational domain even if the streamwise dimension of the computational domain is extended up to very large scale (see Komminaho, Lundbladh & Johansson 1996). Papavassiliou & Hanratty (1997) suggested that streaky structures in plane Couette flow are produced by streamwise-oriented large-scale circulations. Although their simulations were performed at $Re_\tau = 157$, and thus not high enough to separate the near-wall and outer regions, large-scale circulations could be associated with an inverse cascade, i.e. some merging processes of structures in three-dimensional wall-bounded turbulent flows. In this paper, we define a large-scale structure to be the flow consisting of a pair of streamwise oriented large-scale circulations with a low-speed streaky structure between them in the outer region. Large-scale structures have also been observed in plane Poiseuille flow in statistical studies and visualization, although they are slightly shorter, i.e. more three-dimensional, than those in Couette flow. However, the spanwise extent of the large-scale structure in any case is not large, and is thought to lie between a factor of 1.3 and 2 times half the channel width according to Abe, Kawamura & Matsuo (2001) and Jiménez (1998).

Although the self-sustaining process may largely account for turbulent fluctuations in the near-wall region, the minimal flow unit is a relatively small subspace of the huge computational domains which are required to simulate real turbulence. In real turbulence, therefore, a huge number of these near-wall structures, interacting and developing spatially, participate in the production and transfer of turbulent fluctuation toward the outer region. Our main interest is in understanding the dynamical behaviour of a group of near-wall structures, and investigating whether this could be associated with large-scale structures. As a first step, we prevent the near-wall and large-scale structures from evolving spatially in the streamwise direction and thus restrict the streamwise length of the computational box to the minimal length in DNS of channel flow. Because the channel has a relatively large spanwise extent, we will call this box a ‘streamwise-minimal’ channel. Furthermore, the dynamical behaviour of a group of near-wall structures is hereinafter referred to as the collective motion of wall streaks. In the streamwise-minimal channel-flow turbulence, we also find a large-scale structure in the outer region, which closely resembles those observed in huge domains. Our large-scale structure seems to couple tightly with the collective motion of wall streaks and be sustained autonomously as a whole.

Del Álamo & Jiménez (2001, 2003) reported that the pre-multiplied power spectrum of the streamwise velocity, which characterizes the large-scale structure, decomposes into two components: quasi-isotropic modes of relatively short (but long enough) streamwise length scales and long deep modes. The latter modes are extremely long (maybe infinite) in the streamwise direction and penetrate deeply into the near-wall region. They suggested that the latter modes interact with the near-wall region. It will be seen that our large-scale structures found in the present study in streamwise-minimal channels are dominated by modes with $k_x = 0$, i.e. of infinitely long streamwise length. We will conclude, therefore, that our large-scale structure corresponds to their long deep modes and thus reflects some properties of the large-scale structure observed in real turbulence. Thus, since the streamwise-minimal channel may accommodate a large-scale structure, the collective motion of wall streaks and their interactions as well as the self-sustaining processes of individual near-wall structures, the studies contribute significantly to the elucidation of real wall-bounded turbulence.

In the present approach, after some comments on the numerical method and parameters used in the present work, we will show that large-scale structures can exist even in the streamwise-minimal channel used in this study. We will then propose a mechanism for the sustenance of a large-scale structure focusing on the interaction between a large-scale structure and near-wall structures. In the mechanism proposed, the large-scale structure directly interacts with the near-wall region, without any intermediate processes. It should, of course, be noted that our proposed mechanism is just one possibility. Since our channel is a subspace involved in huge domains, other mechanisms, such as that proposed by Adrian, Meinhart & Tomkins (2000) in which a hierarchy of horseshoe vortices with large streamwise-scale constructs a large-scale structure, may also be important in more realistic cases.

2. Streamwise-minimal channel

The numerical scheme we used to simulate channel flow is the same as used in Itano & Toh (2001), which is based on that of Kim, Moin & Moser (1987). The origin of the coordinate system is taken on the midplane of the channel with the x, y and z axes in the streamwise, wall-normal and spanwise directions, respectively. The no-slip boundary condition is imposed at the top ($y = +h$) and bottom ($y = -h$) walls, where h is half the channel width. The flow is driven by constant streamwise volume flux per unit spanwise length Q. We define the characteristic velocity U_c as $3Q/4h$; for laminar Poiseuille flow, U_c is just the centreline velocity. In the present work, we fix the Reynolds number based on U_c, h and the kinematic viscosity ν at 9000 and 3000. In order to guarantee the total computational time, T, is long enough for the statistical convergence of turbulent flow in our domain, we confirmed that the relative error of the time average of wall friction defined as $|\langle f\rangle_t - \langle f\rangle_T|/\langle f\rangle_T$ is less than 0.5% for any $t > T/2$, where

$$\langle f\rangle_t = \frac{1}{t}\int_0^t \nu\frac{\partial U}{\partial y}(t', y = \pm h)\,\mathrm{d}t'.$$

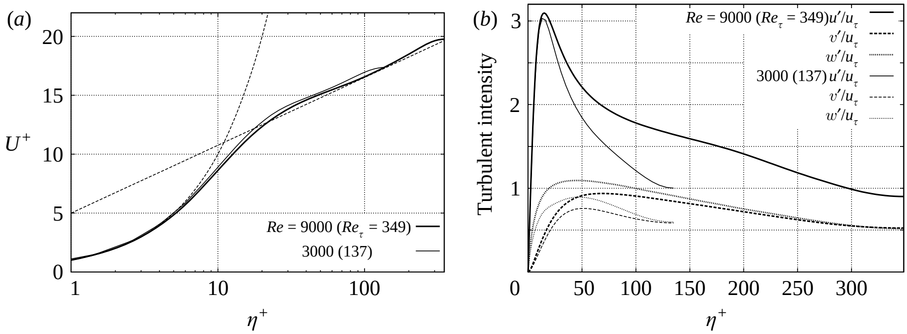

FIGURE 1. (*a*) Mean streamwise velocity of the present channels in wall units. Dashed lines are $U^+ = 2.5\log\eta^+ + 5$ and $U^+ = \eta^+$, where η is the distance from the wall, $\eta = h - |y|$. (*b*) Turbulent intensity profiles. Thick curves correspond to streamwise (solid), wall-normal (dashed), and spanwise (dotted) velocity profiles of $Re = 9000$ ($Re_\tau = 349$). Thin curves correspond to those of $Re = 3000$ ($Re_\tau = 137$).

		Re	Re_τ	L_x^+	L_z^+
Minimal flow	Jiménez & Moin (1991)	2000		(300)	(100)
		3000		(250)	(110)
		5000		(400)	(80)
	Jiménez & Pinelli (1999)	4500	201	360	105
		9000	428	448	128
		18 000	633	397	113
Large-scale structure	Moser *et al.* (1999)		180	2270	756
			395	2560	1250
			590	3720	1840
	Del Álamo & Jiménez (2001)		185	6974	2320
			550	13 800	6910
	Abe, Kawamura & Matsuo (2001)		640	4020	12 100
Present work		9000	349	384	833
		3000	137	259	518

TABLE 1. Parameters used in several past studies.

The Reynolds number based on the friction velocity, $Re_\tau = U_\tau h/\nu$, where $U_\tau = \sqrt{\langle f\rangle_T}$, is 349 and 137 for cases of $Re = 9000$ and 3000, respectively.

As seen in figure 1(*a*), the empirical law of the wall and log-law velocity profile whose description has been obtained by many workers (for example, see Schlichting 1979), both appear to offer good approximations for both cases of $Re = 9000$ and 3000. However, in the $Re = 3000$ case, Re_τ is less than 180, the critical Reynolds number at which turbulent channel flow is free of the obvious low-Reynolds-number effect pointed out by Moser, Kim & Mansour (1999). We can see that the apparent log-law in the $Re = 3000$ case has a larger intercept than in the $Re = 9000$ case. Moreover, as seen in the next section, a large-scale structure is not remarkably distinct from near-wall structures in the $Re = 3000$ case. These suggest that the Reynolds number seems to be slightly too small for the outer region to develop sufficiently.

Table 1 summarizes the Reynolds numbers and dimensions of computational domains which were used in several earlier studies. In the present work, the streamwise length of our channel is set to be approximately minimal by reference to the domain sizes of the minimal flow units used by Jiménez & Moin (1991) and Jiménez & Pinelli (1999). Thus, it is obvious that the streamwise length of our domain, L_x, is much shorter than that used in the earlier studies which suggested the existence of a large-scale structure in the turbulent channel flow.

On the other hand, the spanwise extent of the domain is relatively wide; since it is more than 500 wall units, at least more than five wall streaks could survive in the near-wall region in our domain. The spanwise extent of the domain, L_z, is $2.39h$ and $3.78h$ for $Re = 9000$ and 3000, respectively. In both cases, L_z exceeds the critical value, $2h$, necessary for a large-scale structure to exist in the outer region, as described by Jiménez (1998).

Figure 1(*b*) shows the turbulent intensity obtained for our channel. Note that the peak value of the turbulent intensity of streamwise velocity fluctuation is somewhat larger than that obtained from DNS with huge domains; for example, Moser *et al.* (1999) found turbulent intensity to vary between 2.6 and 2.8 for $180 < Re_\tau < 590$. This difference also exists in comparison with the minimal flow as described by Jiménez & Pinelli (1999). Thus, the large peak value is probably due to our domain size; the large spanwise extent of our domain allows large-scale structures to exist in the outer region, while the short streamwise length of our domain may encourage increased interaction between near-wall structures and large-scale structures.

In fact, turbulent intensity in the centre region of our domain lies between those obtained for the minimal flow unit and huge domains, especially regarding the intensity of the spanwise velocity. This suggests that our channel contains something other than near-wall structures, although our channel cannot completely reproduce turbulent flow in huge domains.

3. Large-scale structure

The pre-multiplied power spectra have been often used to suggest the existence of large-scale structures in channel flow, e.g. Jiménez (1998) or Abe *et al.* (2001). Specifically, we define the pre-multiplied power spectra as follows:

$$\phi_{ff}(k_z)|_\eta = k_z E_{ff}(k_z, y) \Bigg/ \sum_{k_z=2\pi/L_z}^{max(k_z)} E_{ff}(k_z, y),$$

$$E_{ff}(k_z, y) = \frac{1}{T L_x} \int_0^T \int_0^{L_x} (|f_{k_z}(x, y, t)|^2 + |f_{-k_z}(x, y, t)|^2)\, \mathrm{d}x\, \mathrm{d}t,$$

where $f_{k_z}(x, y, t)$ is the Fourier coefficient for a spanwise wavenumber k_z of velocity component $f(x, y, z, t)$, $f = u, v, w$ and distance from the wall $\eta = h - |y|$. We use pre-multiplied spectra $\phi(k_z) \equiv k_z E(k_z)$ so that areas under the curve in log–linear plots correspond to the actual energy content, i.e. $E(k_z)\,\mathrm{d}k_z = \phi(k_z)\,\mathrm{d}(\ln k_z)$. Figure 2 shows pre-multiplied power spectra obtained for the present streamwise-minimal channels. The characteristic length giving the spectrum peak is dependent on distance from the wall and is thought to correspond to the spanwise scale of a relatively dominant structure at each distance in the flow.

In both the cases, $Re = 9000$ and 3000, ϕ_{uu} and ϕ_{ww} at $\eta^+ = 5$ peak at approximately $\lambda_z^+ = 100$, which corresponds to the accepted mean spacing of the wall streaks in the near-wall region. With increasing η in both cases, the peaks move to a longer wavelength corresponding to the outer length. However, in the $Re = 3000$ case, the value of the spectral peak monotonically decreases with increasing η. This suggests

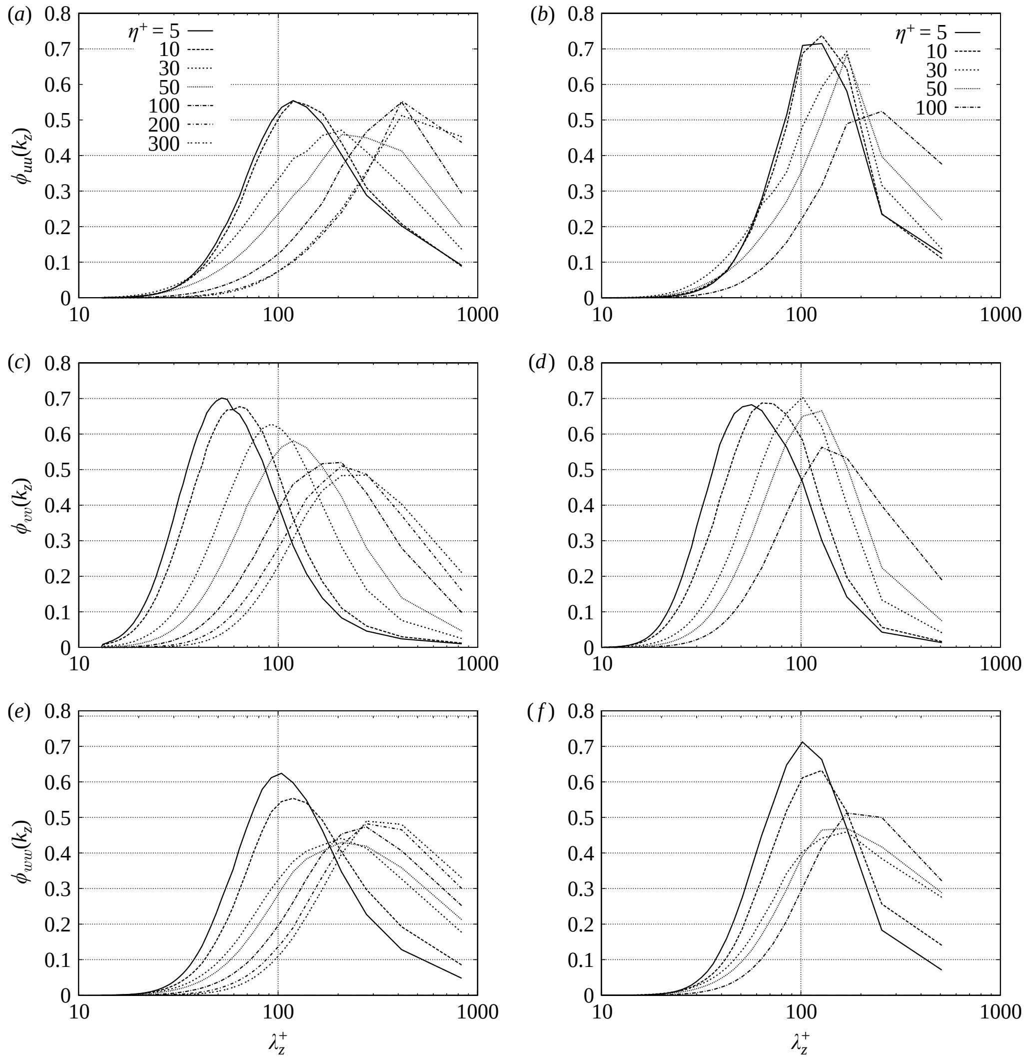

FIGURE 2. Pre-multiplied power spectrum, $\phi(k_z)$, as a function of $\lambda_z = 2\pi/k_z$. (*a*) and (*b*), ϕ_{uu}; (*c*) and (*d*), ϕ_{vv}; (*e*) and (*f*), ϕ_{ww}. (*a*), (*c*) and (*e*), for $Re = 9000$ ($Re_\tau = 349$); (*b*), (*d*) and (*f*), for $Re = 3000$ ($Re_\tau = 137$). In both cases, increasing distance from the wall, $\eta = h - |y|$, corresponds to a rightward shift towards the long-wavelength end of the spectrum. All the spectra are normalized to unit area under the curve in the log–linear plots, to emphasize their frequency content.

that, for this case, the large-scale structure has not separated from the near-wall structures, that is, there is an insufficient development of the outer region in this case.

Compared with the spectra of u and w, the spanwise wavelength of the peak of ϕ_{vv} is half of the other two, in the near-wall region; this was considered by Kim *et al.* (1987). They investigated the spanwise two-point autocorrelations of velocity components and found that the minimum of the spanwise autocorrelation $R_{vv}(z)$ of wall-normal velocity v is at $\Delta z^+ \simeq 25$ which is half of the separation of that of the streamwise velocity. They claimed that this spanwise characteristic separation of $R_{vv}(z)$ is consistent with the mean diameter of the streamwise vortical structures.

Thus, it is not surprising that the spanwise length of the peak of ϕ_{vv} differs from that of the near-wall structure; we argue that this is a direct consequence of no-slip and incompressibility in the following. Suppose that the flow near the wall is independent of the streamwise direction and can be described as

$$v(\eta, z) \sim (v_1 \mathrm{e}^{\mathrm{i}\kappa z} + v_2 \mathrm{e}^{\mathrm{i}2\kappa z})\eta^2 \quad \text{and} \quad w(\eta, z) \sim (w_1 \mathrm{e}^{\mathrm{i}\kappa z} + w_2 \mathrm{e}^{\mathrm{i}2\kappa z})\eta, \tag{3.1}$$

where $\lambda_z = 2\pi/\kappa$ is taken as the most energetic spanwise wavelength in the near-wall region, $\lambda_z^+ = 100$ in wall units ($\lambda_z^+ = \lambda_z U_\tau/\nu$). Exploiting the incompressibility, we obtain $\phi_{vv}(\kappa)/\phi_{vv}(2\kappa) \sim \kappa|v_1|^2/(2\kappa|v_2|^2) = \kappa|\kappa w_1/2|^2/(2\kappa|2\kappa w_2/2|^2) = |w_1/w_2|^2/8$ and $\phi_{vv}(\kappa)/\phi_{vv}(2\kappa) \sim \phi_{ww}(\kappa)/\phi_{ww}(2\kappa)/4$ because $\phi_{ww}(\kappa)/\phi_{ww}(2\kappa) \sim |w_1/w_2|^2/2$. Since $\phi_{ww}(\kappa)/\phi_{ww}(2\kappa) \approx 3$ is obtained from figure 2(*e*) and thus $\phi_{vv}(2\kappa) > \phi_{vv}(\kappa)$, we find that $\phi_{ww}(k_z)$ peaks at $\lambda_z^+ = 100$, but $\phi_{vv}(k_z)$ instead peaks at 50. (In fact, comparing results with the full numerical solution we were able to see that the two-mode approximations, (3.1), appear to adequately capture the leading-order behaviour of the solutions.)

In this section, we have shown that the pre-multiplied power spectra of streamwise and spanwise velocities have two specified peaks corresponding to the mean intervals of near-wall structure in the near-wall region and large-scale structure in the outer region. These characteristics have been also reported in many studies using DNS with a more realistic huge channel, for example Jiménez (1998). The similarities between the streamwise-minimal channel and a huge channel suggest that the former contains a large-scale structure quite close to that in the latter. If this is so, we will then be interested in what makes a large-scale structure and, how the large-scale structure contributes to turbulence, in the streamwise-minimal channel.

4. Co-supporting cycle

The streamwise-minimal channel allows for only one near-wall and one large-scale structure with respect to the streamwise direction. This artificial restriction inhibits some of the rich spatio-temporal properties observed in huge domains, for example, the spatial growth of structures and the interaction between structures aligned in the streamwise direction (Adrian *et al.* 2000). Nevertheless, this simplified system still appears to include fundamental dynamics of both the large-scale structure and near-wall structures.

The structures interact with each other while moving around in a streamwise cross-section and repeating their own dynamical processes. Here, in order to understand the dynamics of the structures as a whole, we represent the location of a near-wall structure or a large-scale structure by the spanwise position of local minima of the streamwise velocity, $\zeta(t, \eta^+)$, at $\eta^+ = 5$ (near-wall region) or 200 (outer region), respectively. The position ζ is defined as follows:

$$\frac{\partial u^{2D}}{\partial z}(t, y, z) = 0, \quad \frac{\partial^2 u^{2D}}{\partial z^2}(t, y, z) > 0, \quad u^{2D}(t, y, z) < U(y) \quad \text{at} \quad z = \zeta(t, \eta^+),$$

where

$$u^{2D}(t, y, z) = \frac{1}{L_x}\int_0^{L_x} u(t, x, y, z)\,\mathrm{d}x, \quad U(y) = \frac{1}{TL_z}\int_0^T\int_0^{L_z} u^{2D}(t, y, z)\,\mathrm{d}z\,\mathrm{d}t.$$

Hereinafter, the area in the (y, z)-plane satisfying $u^{2D} < U$ ($u^{2D} > U$) is called the low-speed (high-speed) zone. In figure 3, we plot time evolution of the spanwise location of all the local minima in the near-wall and outer regions, which allows us to trace the spanwise movement and generation processes of both near-wall and

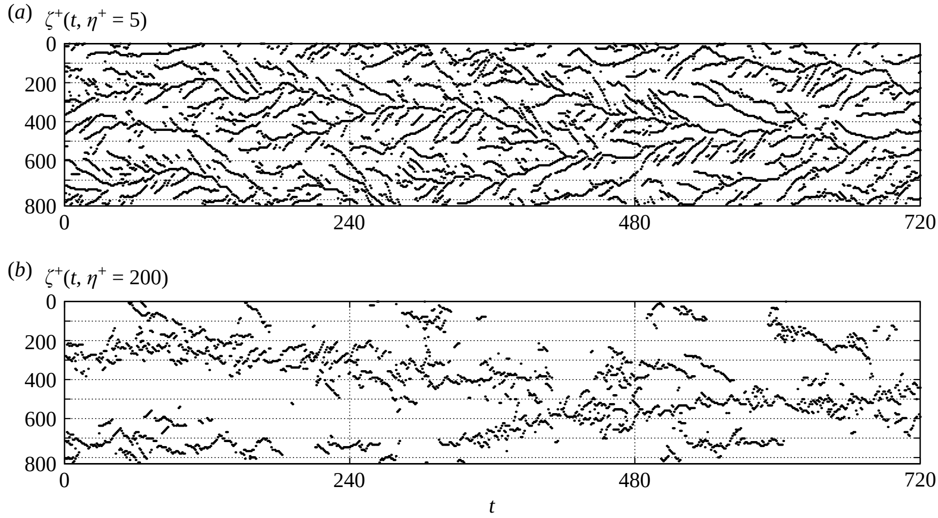

FIGURE 3. The time evolution of the spanwise locations of low-speed regions, which are identified as points satisfying $\partial u^{2D}/\partial z = 0$ and $\partial^2 u^{2D}/\partial z^2 > 0$ in the low-speed zone $u^{2D} < U$. (*a*) the near-wall region, $\eta^+ = 5$, (*b*) in the outer region, $\eta^+ = 200$, in the lower half domain ($-h < y < 0$) in the $Re = 9000$ ($Re_\tau = 349$) case.

large-scale structures. In fact, the characteristic spanwise wavelength λ_z^+ at the peak of ϕ_{uu} mentioned in the previous section finds reasonable agreement with the mean spanwise interval between two adjacent minima; as may be seen from the figure, $\Delta\zeta^+ \approx 100$ and 400 for $\eta^+ = 5$ and 200, respectively.

Here we adopted the ancillary condition $u^{2D}(t, y, z) < U(y)$ to pick out low-speed zones. Even in the high-speed zones, local minima can exist. However, such local minima are not clear curves corresponding to near-wall structures, but scattered points or very short segments as shown in figure 7 in the Appendix. This means that, in the high-speed zones, dominant or matured near-wall structures rarely exist. Nucleation of another near-wall structure by the splitting of a near-wall structure seems to occur but is rare. Therefore the main mechanism of nucleation of near-wall structures seems to be some instability.

From figure 3(*a*), we can see that the branches may be classified into two types: dominant branches, which survive for a relatively long time, and weak branches. As time elapses, weak branches are successively merged into a few dominant branches in almost all of the merging events in the near-wall region, while some weak branches emerge from structure-free areas. The branches in figure 3(*a*) are reminiscent of rivers in a map of a mountainous area and thus we call the region where branches gather a 'valley' and the structure-free region where branches emerge a 'watershed'. It should be noted that a group of loci of minima of u in figure 3(*b*) appears to be located always above a long-lived branch in figure 3(*a*), while a watershed in figure 3(*a*) separates two adjacent groups of loci of minima of u in figure 3(*b*). In addition, it is worth remarking there is apparently some scale similarity between figures 3(*a*) and 3(*b*), if we regard groups of loci of minima in figure 3(*b*) as curves. Note that in the near-wall region, each branch denotes a single near-wall structure, while in the outer region, a group of loci of minima of u represents a single large-scale structure. The dynamics of the large-scale structures and near-wall structures in the streamwise-minimal channel may be described as follows (see figure 4). Immature near-wall structures

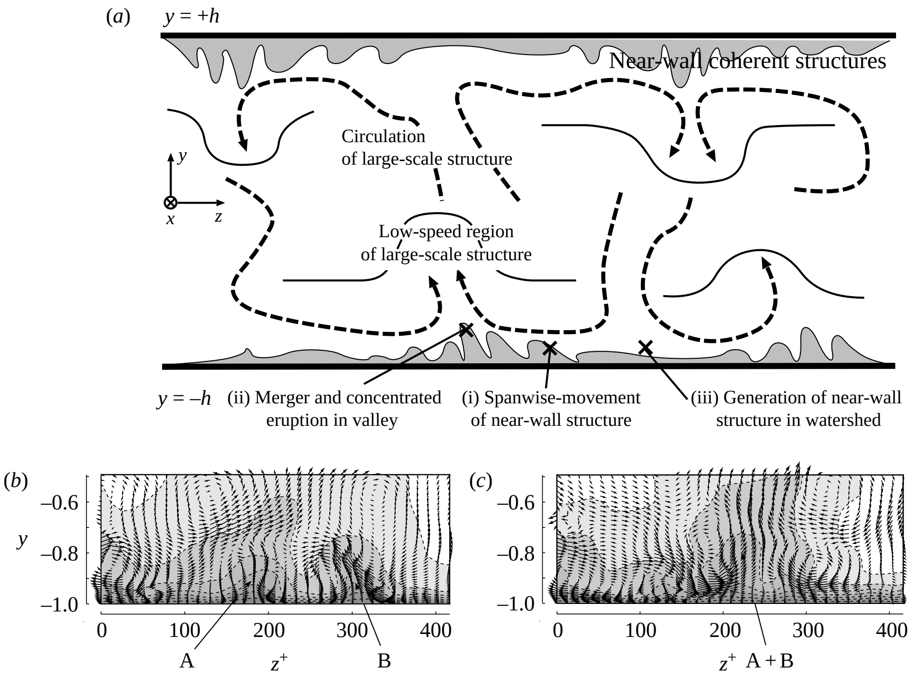

FIGURE 4. (*a*) Schematic view of a snapshot in our channel in a *z*–*y* cross-section, where three elementary processes of the co-supporting cycle are described. Thin solid curves indicate contours of u^{2D} in the outer region, each bulge of which corresponds to the low-speed region of a large-scale structure. The circulation of a large-scale structure is represented by thick dashed curve. Shaded regions near the walls denote wall streaks. (*b*), (*c*) Snapshots of flow in the $Re = 9000$ ($Re_\tau = 349$) case in a *z*–*y* cross-section at $t = 120$ and 140, respectively. The vector field indicates (w^{2D}, v^{2D}). The shaded region indicates $u^{2D} < 0.7U_c$ and contour levels are $0.6U_c, 0.5U_c, \ldots, 0.1U_c$. A concentrated eruption follows a merging event of near-wall structures A and B in (*b*).

are continually generated through a local instability near a watershed between two adjacent large-scale circulations and slowly move toward either of the two. Moreover, a dominant near-wall structure continually attracts and merges weaker structures into itself, beneath the low-speed region of a large-scale structure. These facts suggest a tight coupling between a large-scale structure and near-wall structures, which consists of three elementary processes described below (see figure 4*a*).

(i) One of the two circulations of a large-scale structure induces the near-wall structures to move in the spanwise direction toward the area under the low-speed region of the large-scale structure.

(ii) Generally, when two near-wall structures merge, a concentrated eruption† occurs which causes an influx of fluid from the near-wall region into the outer region. Such a merging event is seen in figures 4(*b*) and 4(*c*). The spanwise location of these eruptions tends to be reasonably immobile. In our simulations, a concentrated eruption appears to be stronger than a burst which occurs as part of the self-sustaining

† The term ‘eruption’ is used for blowup occurring through the merging process of near-wall structures and ‘burst’ for blowup as a part of SSP of a single near-wall structure. The eruption often occurs and thus is robust. This may be confirmed in some animations at the web: http://www-kyoryu.scphys.kyoto-u.ac.jp/movies/.

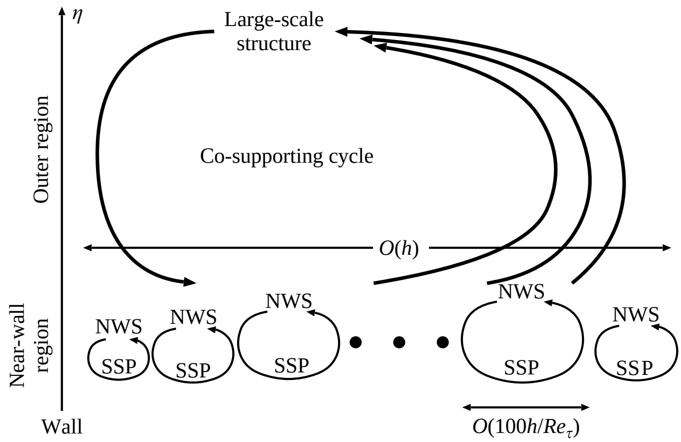

FIGURE 5. Co-supporting cycle of a large-scale structure and near-wall structures. SSP, self-sustaining process; NWS, near-wall structure.

process of a near-wall structure. The concentrated eruption also brings about strong suction from both sides which acts to maintain the large-scale circulation, moreover, through drawing in fluid with lower streamwise speed from the near-wall region it also acts to maintain the low-speed region of the large-scale structure.

(iii) There is a watershed, a separation region close to the wall formed by the part of one of the circulations of the large-scale structure, in which fluid is directed towards the channel wall. In these regions, new wall streaks are created continually through some instability.

The coupling with the near-wall structures through the three processes above probably enables the large-scale structure to survive (see figure 5). However, in the cycle, the large-scale structure is not just passive, but active enough to contribute to the generation, spanwise-movement and merger of near-wall structures, which re-activate the large-scale structure itself. Therefore, we denote the whole cycle as a 'co-supporting cycle' of a large-scale structure and near-wall structures, to distinguish this from the self-sustaining process of a single near-wall structure.

Some further comments should be made regarding this co-supporting cycle. First, a sequence of eruptions in the cycle tends to stay around a fixed position in z for a relatively long time in comparison with the period of the self-sustaining process. To quantify the immobility of large-scale structures, we introduce an autocorrelation function of velocity components and then define the correlation time as follows:

$$C[g;y,\tau]=\frac{\langle\hat{g}(y,z,t)\hat{g}(y,z,t+\tau)\rangle}{\langle\hat{g}(y,z,t)^2\rangle}, \tag{4.1}$$

where g is one of velocity components u, v and w,

$$\hat{g}(y,z,t)=g^{2D}-\langle g\rangle,\quad g^{2D}=\frac{1}{L_x}\int_0^{L_x}g(x,y,z,t)\,\mathrm{d}x$$

and

$$\langle G\rangle=\frac{1}{TL_xL_z}\int_0^T\int_0^{L_x}\int_0^{L_z}G(x,y,z,t)\,\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}t$$

for an arbitrary function G. The average time T taken here is sufficiently large so that $C[g;y,\tau]$ converges. With $C[g;y,\tau]$ the correlation time $T^g(\eta)$ of g at distance

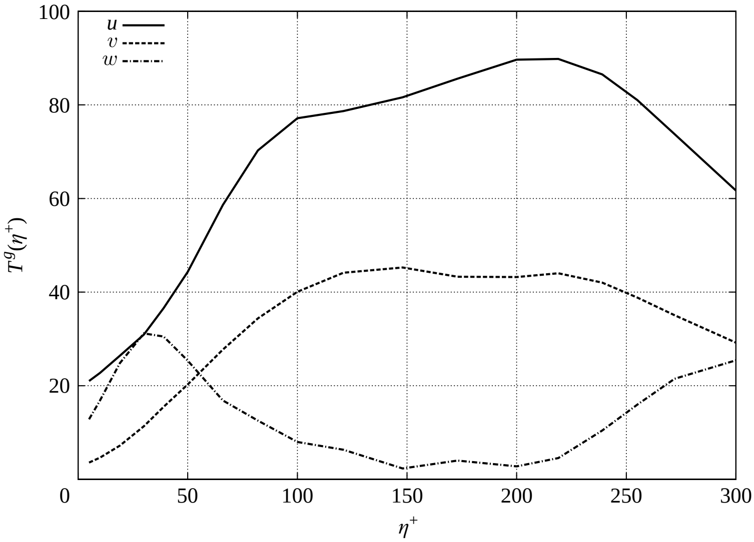

FIGURE 6. Correlation time of velocity against distance from the wall in wall units is obtained in the $Re = 9000$ ($Re_\tau = 349$) case. u (solid), v (dashed), w (dash-dotted).

η from the wall is defined as

$$T^g(\eta) = \int_0^{\tau_0} C[g; y, \tau]\,\mathrm{d}\tau, \tag{4.2}$$

where τ_0 is the smallest τ satisfying $C[g; y, \tau] = 0.05$, and $\eta = h - |y|$. Figure 6 shows $T^g(\eta)$ for $g = u$, v and w. $T^u(\eta)$ achieves the maximal correlation time around $\eta^+ \approx 200$, which suggests large-scale structures are more immobile in the spanwise direction than near-wall structures. It should also be noted that $T^v(\eta)$ seems to be similar to $T^u(\eta)$. This is consistent with our observation in which large-scale structures are sustained by eruptions from the near-wall region.

Secondly, the spanwise extent of the low-speed region of a large-scale structure is of the order of the spanwise extent in which the suction caused by the eruption influences the near-wall structures. Hence, the circulation of a large-scale structure described in the co-supporting cycle does not necessarily cover the entire channel width, for example circulations shown around the right-hand corner in figure 4(*a*).

5. Concluding remarks

In this paper, we have shown that large-scale structures exist even in a streamwise-minimal box whose streamwise dimension is confined to the minimal length required for the sustenance of turbulence. The large-scale structure of the present study involves two counter-rotating large-scale circulations and a streak-like low-speed region. Large-scale structures are coupled tightly with near-wall structures and sustained by their direct interaction. We have therefore called this sustaining process of a large-scale structure a co-supporting cycle.

We investigated the time evolutions of the large-scale structures and the collective motion of wall streaks by projecting them onto the streamwise cross-section, because the streamwise-minimal box inhibits their spatial evolutions in the streamwise direction. In the near-wall region, more than five wall streaks can exist simultaneously and each of them repeats the self-sustaining cycle (of the order of 10 unit times) individually like a single wall streak in the minimal flow unit at low Reynolds number. As seen in figure 3(*a*), near-wall structures move around in the spanwise direction and form the valley and watershed structure as a whole owing to some local instability of wall streaks.

When an interval between adjacent (low-speed) wall streaks of near-wall structures exceeds some critical length of the order of 100 wall units, a new wall streak tends to emerge; inversely, a weak wall streak tends to be merged into a dominant wall streak when the interval becomes shorter than another critical length, also around 100 wall units.

This collective motion of wall streaks is coupled directly with large-scale structures without any intermediate structure. In fact, both the nucleation and annihilation of near-wall structures are linked with large-scale circulations. Near-wall structures are created around watersheds and carried sideways into valleys by large-scale circulations; matured near-wall structures die away in the valley bottom by merging with each other and feeding the large-scale circulations in turn.

The near-wall structures gathered around a valley by a large-scale structure are regarded as a spanwise modulation in an array of near-wall structures. We believe that the array of near-wall structures may correspond to a spatio-temporally periodic solution which would be connected to periodic solutions or travelling-wave solutions obtained in a minimal flow unit at low Reynolds numbers. We could imagine that this spanwise modulation developing into a large-scale eruption is triggered by some modulational instability of an array of near-wall structures and then the developed eruption directly drives large-scale structures.

In the co-supporting cycle, large-scale structures and near-wall structures interact directly with each other and sustain themselves. In our simulations, though large-scale structures exist, the value of Reynolds number used is not so high that the scale separation between large-scale structures and near-wall structures is significant. This weak scale separation might allow the direct coupling between them without intermediate processes. As a generation mechanism of large-scale structures, 'inverse cascade' is often referred to because it is based on local interactions between adjacent different scales and thus holds even at high Reynolds numbers. The 'inverse cascade', however, should occur both in scale and in space, i.e. turbulence fluctuation produced around the buffer layer should be transferred into the centre region. In addition, through this 'inverse cascade' energy is not conserved because at each height energy is transferred toward the dissipation range via normal cascade. A concrete mechanism satisfying these features has not been proposed so far. The co-supporting cycle might not be the main generation mechanism of large-scale structure in real turbulence, but it may perhaps be expected to perform an important role.

We restricted the streamwise extent of the box to the minimal. Under this restriction and the periodic boundary condition, large-scale structures, which otherwise grow spatially and are advected faster than near-wall structures, are forced to continue to interact with the near-wall structures that have much shorter time and length scales than large-scale structures. The enhancement of turbulent intensity observed around the buffer layer is probably caused by this strong coupling. This situation seems to be artificial, however, as reported by Del Álamo & Jiménez (2003), the long deep mode, which is extremely long or even infinite in x, interacts with the near-wall region in huge computational boxes. In the streamwise-minimal box, large-scale structures are dominated by the two-dimensional modes with $k_x = 0$, and these modes must correspond to the long deep mode. Therefore, the co-supporting cycle is more realistic than expected.

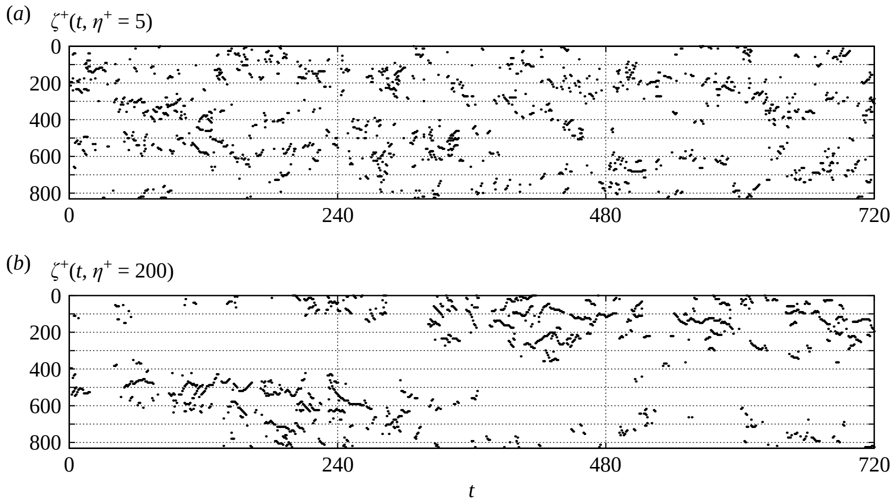

FIGURE 7. The time evolution of the spanwise locations of low-speed regions in the high-speed zone $u^{2D} > U$. The locations are identified as points satisfying $\partial u^{2D}/\partial z = 0$ and $\partial^2 u^{2D}/\partial z^2 > 0$. (*a*) The near-wall region, $\eta^+ = 5$, (*b*) in the outer region, $\eta^+ = 200$, in the lower half domain ($0 < z < 0.5L_z$ and $-h < y < -0.5h$) in the $Re = 9000$ ($Re_\tau = 349$) case. ζ^+ (y-axis) is marked with 100 wall units.

The relatively short spanwise extent of the present computational domain, which is much longer than the mean spacing of the wall streaks, is minimal for the existence of large-scale structures. Thus, this spanwise restriction also limits the spanwise motion of large-scale structures in the same manner as the minimal flow unit prevents near-wall structures from moving in the spanwise direction. This seems to lead to the success in extracting the co-supporting cycle.

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Appendix. Structures in high-speed zones

On the basis of the observation of the time evolution of the spanwise locations of low-speed regions, we have proposed the co-supporting cycle. The locations have been identified as local minimum points of u^{2D} in low-speed zones $u^{2D} < U$. We would expect the ancillary condition $u^{2D} > U$ to exclude a certain structure standing underneath a high-speed zone so that we could conclude that near-wall structures are generated from the 'structure-free' regions. Figures 7(*a*) and 7(*b*) are the counterparts of figures 3(*a*) and 3(*b*), respectively. In these figures, the pattern of generation and spanwise movement of the near-wall structures in high-speed zones $u^{2D} > U$ is less coherent than that in low-speed zones $u^{2D} < U$.

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